

Hypercolor tower

M.A. Zubkov

ITEP, B.Chermushkinskaya 25, Moscow, 117259, Russia and
Moscow Institute of Physics and Technology, 141700, Dolgoprudnyi, Moscow Region, Russia.

We construct the ultraviolet completion of the Standard Model that contains an infinite sequence of Hypercolor gauge groups. So, the whole gauge group of the theory is $\dots \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$. Here $SU(4)$ is the Technicolor group of Farhi - Susskind model. The breakdown of chiral symmetry due to the Technicolor gives rise to finite W and Z boson masses in a usual way. The other Hypercolor groups are not confining. We suggest the hypothesis that the fermion masses are not related in any way to technicolor gauge group. We suppose that the fermion mass formation mechanism is related to the energies much higher than the technicolor scale. Formally the fermion masses appear in our model as an external input. In the construction of the theory we use essentially the requirement that it possesses an additional discrete symmetry \mathcal{Z} that is the continuation of the Z_6 symmetry of the Standard Model. It has been found that there exists such a choice of the hypercharges of the fermions that the chiral anomaly is absent while the symmetry \mathcal{Z} is preserved.

PACS numbers: 12.15.-y, 11.15.Ha, 12.10.Dm

I. INTRODUCTION

Thinking about the possible ultraviolet completion of the Standard Model, we encounter Technicolor and Extended Technicolor theories, Little Higgs models, supersymmetry, extra dimensions, and TeV - scale gravity (see, for example, [1, 2, 3, 4, 5, 6]). However, basing on the present data we cannot make a definite choice. Probably, the data of LHC coming soon will help more.

It is worth mentioning that the Standard Model itself cannot describe physics at the energies above 1 TeV. The conventional way to explain this is based on the concept of "naturalness" and is related to the treatment of the fine tuning of Higgs sector mass parameter as unnatural [7]. Besides, it was shown recently, that the Standard Model in lattice regularization cannot have in principle the value of the ultraviolet cutoff larger than about 1 TeV [8, 9].

In this paper we suggest the model that is based on the ideas of Technicolor. However, our approach differs essentially from the conventional Technicolor approach. The main difference is that we do not incorporate to the theory the Extended Technicolor interactions. That's why the Technicolor interaction serves only as a source of Electroweak gauge symmetry breaking. As for the fermion masses, we suggest the following hypothesis: *the source of the fermion masses is not related to Technicolor interactions and to the transformation of the technifermions to the other physical states*. We suppose that the fermion mass formation mechanism may be related to the energies much higher than it is usually implied in the Extended Technicolor models. The second important supposition of our paper is that the ultraviolet completion of the Standard Model contains an infinite sequence of the gauge groups $\dots \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$. We call this sequence *Hypercolor Tower*, the groups $SU(N)$ of this sequence for $N > 3$ are called hypercolor groups. The Electroweak symmetry breaking occurs due to the $SU(4)$ subgroup. That's why we refer to $SU(4)$ as to the Technicolor group.

Thus we refuse to accept the conventional Extended Technicolor scheme. In spite of this below we briefly remind this scheme and its difficulties. For the review see [1, 2] and references therein. In the Technicolor theory the new Nonabelian gauge interaction is added with the scale $\Lambda_{TC} \sim 1$ TeV, where Λ_{TC} is the analogue of Λ_{QCD} . This new interaction is called Technicolor. The correspondent new fermions are called technifermions. The Electroweak gauge group acts on the technifermions. Therefore, breaking of the chiral symmetry in Technicolor theory causes

Electroweak symmetry breaking. This makes three of the four Electroweak gauge bosons massive. However, pure Technicolor theory cannot explain the appearance of fermion masses.

Usually in order to make Standard Model fermions massive extra gauge interaction is added, which is called Extended Technicolor (ETC) [1, 2]. In this gauge theory the Standard Model fermions and technifermions enter the same representation of the Extended Technicolor group. Standard Model fermions become massive because they may be transformed into technifermions with ejecting of the new massive gauge bosons. Then the quark and lepton masses are evaluated at one loop level as $m_{q,l} \sim \frac{N_{TC}\Lambda_{TC}^3}{\Lambda_{ETC}^2}$, where Λ_{TC} is the Technicolor scale while Λ_{ETC} is the scale of the new strong interaction called Extended Technicolor. (Spontaneous breakdown of Extended Technicolor symmetry gives rise to the mass of the new gauge bosons of the order of Λ_{ETC} .)

The ETC models suffer from extremely large flavor - changing amplitudes and unphysically large contributions to the Electroweak polarization operators [1]. The possible way to overcome these problems is related to the behavior of chiral gauge theories at large number of fermions or for the higher order representations. Namely, the near conformal behavior of the Technicolor model allows to suppress dangerous flavor changing currents as well as to decrease the contribution to S - parameter [10, 11]. However, the generation of t - quark mass in these models still causes serious problems[25].

As it was already mentioned, in the present paper we avoid the mentioned problems specific for the ETC models because we do not require that the fermion masses are related in any way to Technicolor interactions. We suppose, that the chiral symmetry breaking in the Technicolor theory gives rise to the gauge boson masses only. The formation of fermion masses remains out of our model. We only notice here that the fermion masses in relativistic theory is related to the transition amplitude between the right handed and left handed fermions. That's why any process that leads to appearance of such amplitude may be treated as the fermion mass formation mechanism. In particular, we may suppose, that the processes like this happen at extremely high energies, probably, even of the order of Plank mass. So, formation of fermion masses may, in principle, be related to quantum gravity.

In order to incorporate formally fermion masses to the chiral invariant theory we introduce the auxiliary field $\Omega \in SU(2)$. We imply that there is no dynamical term in the action that contains its derivatives. The physical sense of this field is that it picks up the parity partner for each right - handed spinor. At the same time the theory possesses chiral invariance at the level of bare action. In a certain sense Ω plays the role of the usual Higgs field with frozen radius and without dynamical term in the action. The gauge group of the theory is chosen to be the infinite product of $SU(N)$ groups and the gauge group of $SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)/Z_{12}$ Farhi - Susskind Technicolor theory [13].

In order to fix the hypercharge assignment of the model we require that the theory is invariant under the additional discrete \mathcal{Z} symmetry. This symmetry is the continuation of the Z_6 symmetry of the Standard Model [14, 15, 16, 17] to the Hypercolor models [18]. It has been found long time ago, that the spontaneous breakdown of $SU(5)$ symmetry in Grand Unified Theory actually leads to the gauge group $SU(3) \times SU(2) \times U(1)/Z_6$ instead of the conventional $SU(3) \times SU(2) \times U(1)$ (see, for example, [14] and references therein). However, the Z_6 symmetry is not the subject of the $SU(5)$ unification only. Actually, the Z_6 symmetry is present in the Standard Model itself without any relation to the particular Unified theory [15, 16, 17]. The Z_6 symmetry is rather restrictive and it forbids, for example, the appearance of such particles as left - handed Standard Model fermions with zero hypercharge. It was shown in [16], that the Unified models based on the Pati - Salam scheme [19] may possess the Z_6 symmetry. Besides, it was found that in the so - called Petite Unification models (also based on the Pati-Salam scheme) the additional discrete symmetry is present (Z_2 or Z_3 depending on the choice of the model) [17].

The reason of the application of this symmetry to our construction is that we guess the Z_6 symmetry of the Standard Model is not accidental. That's why, we suppose it must emerge in a certain way in the more fundamental theory. Besides, we find that the \mathcal{Z} symmetry has a certain influence on the monopole content of the hypothetical Unified theory that incorporates our Hypercolor tower as a low energy approximation.

The paper is organized as follows. In the 2 - nd section we describe the basic ingredients of our model, i.e. the gauge group and the sequence of fermions. In the third section we introduce parity conjugation of two - component spinors used in our model to incorporate fermion masses. In the 4 - th section we describe the Z_6 symmetry of the Standard Model and the chosen way to continue it to the Hypercolor groups. In the 5 - th section we describe the first element of the sequence of Hypercolor groups, i.e. the Farhi - Susskind Technicolor $SU(4)$ interactions. We explain how the \mathcal{Z} symmetry fixes the hypercharge assignment for technifermions. In the 6 - th section the way to introduce fermion masses to the theory is described. In the 7 - th section the formation of chiral condensates in our model is described. In the 8 - th section we describe the next element in the sequence of Hypercolor groups, i.e. the $SU(5)$ interactions. In the 9 - th section the generalization of our consideration to the Hypercolor groups $SU(N)$ with arbitrary N is explained. In the 10 - th section we discuss the relation between the \mathcal{Z} symmetry and the properties of the hypothetical Unified theory. In the 11 - th section the dynamics of Hypercolor interactions is briefly reviewed. In the 12 - th section we end with our conclusions.

II. THE BASIC INGREDIENTS OF THE MODEL

In our approach the theory contains $U(1)$ gauge group and the groups $SU(N)$ with any N . So, the gauge group of the theory is

$$G = \dots \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}, \quad (1)$$

where \mathcal{Z} is the discrete group to be specified below.

Next, we suppose, that in the theory any fermions are present that belong to the fundamental representations of the $SU(N)$ subgroups of G . So, the possible fermions are right - handed $\Psi_{A,Y}^{\alpha i_{k_N} \dots i_{k_3} i_{k_2}}$ and left - handed $\Theta_{\dot{\beta} A, Y}^{i_{k_N} \dots i_{k_3} i_{k_2}}$, where α and $\dot{\beta}$ are spinor indices, A enumerates generations while index i_k belongs to the subgroup $SU(k)$. Here Y is the $U(1)$ charge of the given fermion. In particular, the fermions $\Psi_{A,Y}$ are present that have no indices and the only subgroup that acts on $\Psi_{A,Y}$ is $U(1)$. Moreover, we suppose that the fermions are present such that G does not act on them at all. We denote them $\Psi_{A,0}$. All fermions in the theory are two - component spinors. We also suppose from the very beginning that the $SU(2)$ group acts on the left - handed spinors only. The action of parity conjugation on them will be considered later. For the simplicity we omit below both spinor and generation indices. So, our fermions are

$$\begin{aligned} U(1) : & \quad \Psi_0, \Psi_{Y_1}, \Psi_{Y'_1}, \dots; \\ U(1), SU(2) : & \quad \Theta_{Y_2}^{i_2}, \Theta_{Y'_2}^{i_2}, \dots; \\ U(1), SU(3) : & \quad \Psi_{Y_3}^{i_3}, \Psi_{Y'_3}^{i_3}, \dots; \\ U(1), SU(2), SU(3) : & \quad \Theta_{Y_{32}}^{i_3 i_2}, \Theta_{Y'_{32}}^{i_3 i_2}, \dots; \\ U(1), SU(4) : & \quad \Psi_{Y_4}^{i_4}, \Psi_{Y'_4}^{i_4}, \dots; \\ U(1), SU(2), SU(4) : & \quad \Theta_{Y_{42}}^{i_4 i_2}, \Theta_{Y'_{42}}^{i_4 i_2}, \dots; \\ U(1), SU(3), SU(4) : & \quad \Psi_{Y_{43}}^{i_4 i_3}, \Psi_{Y'_{43}}^{i_4 i_3}, \dots; \\ U(1), SU(2), SU(3), SU(4) : & \quad \Theta_{Y_{432}}^{i_4 i_3 i_2}, \Theta_{Y'_{432}}^{i_4 i_3 i_2}, \dots; \\ & \quad \dots \end{aligned} \quad (2)$$

Here in each row we list the subgroups of G that act on the fermions listed in the row. In each row the allowed values of $U(1)$ charge are denoted by Y, Y' , etc.

Let us consider the first row. Here in order to reproduce the Standard Model we restrict ourselves by the values of Y equal to 0 and -2 . Next, the second row must contain the only element with $Y = -1$. The third row contains two elements with $Y = \frac{4}{3}$ and $Y = -\frac{2}{3}$. In the forth row we have the only element with $Y = \frac{1}{3}$. This row completes the Standard Model and we enter the rows related to its ultraviolet completion.

Before dealing with these next rows let us describe how parity conjugation of spinors is incorporated in our model. We shall also remind what we call the additional Z_6 symmetry in the Standard Model and how can it be continued to the Hypercolor models.

III. PARITY CONJUGATION

Let us specify how parity conjugation \mathcal{P} acts on the fermions. If only two fermions χ^α and $\eta_{\dot{\alpha}}$ are present, then $\mathcal{P}\chi^\alpha(t, \bar{r}) = i\eta_{\dot{\alpha}}(t, -\bar{r})$; $\mathcal{P}\eta_{\dot{\alpha}}(t, \bar{r}) = i\chi^\alpha(t, -\bar{r})$. In our case we require that for any configuration of $SU(N)$ ($N > 2$) indices there exist two right - handed spinors and one $SU(2)$ doublet. The parity conjugation connects each of the right handed spinors with a component of the $SU(2)$ doublet. Thus

$$\begin{aligned} \mathcal{P}\Psi_0(t, \bar{r}) &= i\Omega_{i_2}^1(t, -\bar{r})\Theta_{-1}^{i_2}(t, -\bar{r}); \mathcal{P}\Psi_{-2} = i\Omega_{i_2}^2\Theta_{-1}^{i_2}; \\ \mathcal{P}\Psi_{\frac{4}{3}}^{i_3} &= i\Omega_{i_2}^1\Theta_{\frac{1}{3}}^{i_3 i_2}; \mathcal{P}\Psi_{-\frac{2}{3}}^{i_3} = i\Omega_{i_2}^2\Theta_{\frac{1}{3}}^{i_3 i_2}; \\ \mathcal{P}\Psi_{Y_4}^{i_4} &= i\Omega_{i_2}^1\Theta_{Y_{42}}^{i_4 i_2}; \mathcal{P}\Psi_{Y'_4}^{i_4} = i\Omega_{i_2}^2\Theta_{Y_{42}}^{i_4 i_2}; \\ \mathcal{P}\Psi_{Y_{43}}^{i_4 i_3} &= i\Omega_{i_2}^1\Theta_{Y_{432}}^{i_4 i_3 i_2}; \mathcal{P}\Psi_{Y'_{43}}^{i_4 i_3} = i\Omega_{i_2}^2\Theta_{Y_{432}}^{i_4 i_3 i_2}; \\ & \quad \dots \end{aligned} \quad (3)$$

Here Ω is an auxiliary $SU(2)$ field. $[\Omega^1]^*$ and $[\Omega^2]^*$ belong to the fundamental representation of $SU(2)$ subgroup of G . $U(1)$ subgroup of G acts on Ω in such a way that Ω^1 has hypercharge 1 while Ω^2 has hypercharge -1 .

Expression (3) means that it is chosen dynamically, which component of Θ is connected via parity conjugation with the given Ψ . The choice of parity conjugated component of Θ is performed using an auxiliary field Ω . The physical sense of this field is that it picks up the parity partner for each right - handed spinor in a way that formally respects the chiral symmetry of the theory.

IV. \mathcal{Z} SYMMETRY

Here we follow the analysis of [16, 17, 18]. Within the Standard Model for any path \mathcal{C} , we may calculate the elementary parallel transporters

$$\begin{aligned}\Gamma &= \text{P exp}(i \int_{\mathcal{C}} C^\mu dx^\mu) \\ U &= \text{P exp}(i \int_{\mathcal{C}} A^\mu dx^\mu) \\ e^{i\theta} &= \exp(i \int_{\mathcal{C}} B^\mu dx^\mu),\end{aligned}\tag{4}$$

where C , A , and B are correspondingly $SU(3)$, $SU(2)$ and $U(1)$ gauge fields of the Standard Model.

The parallel transporter correspondent to each fermion of the Standard Model is the product of the elementary ones listed above. Therefore, the elementary parallel transporters are encountered in the theory only in the following combinations: $e^{-2i\theta}$; $U e^{-i\theta}$; $\Gamma U e^{\frac{1}{3}\theta}$; $\Gamma e^{-\frac{2}{3}\theta}$; $\Gamma e^{\frac{4}{3}\theta}$.

It can be easily seen [16] that *all* the listed combinations are invariant under the following Z_6 transformations:

$$\begin{aligned}U &\rightarrow U e^{i\pi N}, \\ \theta &\rightarrow \theta + \pi N, \\ \Gamma &\rightarrow \Gamma e^{(2\pi i/3)N},\end{aligned}\tag{5}$$

where N is an arbitrary integer number. This symmetry allows to define the Standard Model with the gauge group $SU(3) \times SU(2) \times U(1)/Z_6$ instead of the usual $SU(3) \times SU(2) \times U(1)$.

It is worth mentioning that the additional discrete symmetry is rather restrictive. Namely, for the Standard Model the requirement that the fermion parallel transporters are invariant under Z_6 gives the condition for the choice of the representations that are allowed for the Standard Model fermions. Say, the left - handed $SU(2)$ doublets with zero hypercharge are forbidden.

The nature of the given additional symmetry is related to the centers Z_3 and Z_2 of $SU(3)$ and $SU(2)$. This symmetry connects the centers of $SU(2)$ and $SU(3)$ subgroups of the gauge group. We suggest the following way to continue this symmetry to the Hypercolor extension of the Standard Model.

We connect the center of the Hypercolor group to the centers of $SU(3)$ and $SU(2)$. Let $SU(K)$ be the Hypercolor group. Then the transformation (5) is generalized to [18]

$$\begin{aligned}U &\rightarrow U e^{i\pi N}, \\ \theta &\rightarrow \theta + \pi N, \\ \Gamma &\rightarrow \Gamma e^{(2\pi i/3)N}, \\ \Pi_4 &\rightarrow \Pi_4 e^{(2\pi i/4)N}, \\ \Pi_5 &\rightarrow \Pi_5 e^{(2\pi i/5)N}, \\ \Pi_6 &\rightarrow \Pi_6 e^{(2\pi i/6)N}, \\ &\dots\end{aligned}\tag{6}$$

Here Π_K is the $SU(K)$ parallel transporter. We construct our model in such a way that the parallel transporters correspondent to the new fermions of the theory are invariant under (6). The resulting symmetry is denoted by \mathcal{Z} and enters expression (1).

V. FARHI - SUSSKIND MODEL

Now let us consider the second four rows in (2). We suggest them in the form that represents $SU(4)$ Farhi - Susskind model of Technicolor [13]. In this model the number of fermions is fixed, the $U(1)$ anomaly is absent but

the hypercharge assignment is not fixed. In order to make a choice we apply the continuation of the Z_6 symmetry found in the Standard Model.

We choose the hypercharge assignment here in such a way that:

1. Mass terms for the fermions proportional to $\Psi^+(t, \bar{r})\mathcal{P}\Psi(t, -\bar{r})$ are invariant under Electromagnetic $U(1)$. Therefore

$$\begin{aligned} Y_4 &= Y_{42} + 1; Y'_5 = Y_{42} - 1; \\ Y_{43} &= Y_{432} + 1; Y'_{43} = Y_{432} - 1; \end{aligned} \quad (7)$$

2. Chiral anomaly should vanish. There are several types of the triangle anomaly [20]. Namely, in addition to the anomaly that may appear in the Standard Model there may appear the anomaly of the following types:

$$\begin{aligned} 1) & SU(4) - SU(4) - SU(4) \\ 2) & SU(4) - SU(4) - U(1) \\ 3) & SU(3) - SU(3) - U(1) \\ 4) & SU(2) - SU(2) - U(1) \\ 5) & U(1) - U(1) - U(1) \end{aligned} \quad (8)$$

The anomaly of the first type is absent because the number of left - handed fermions is equal to the number of the right - handed ones while both types of fermions belong to the fundamental representation of $SU(4)$.

The anomaly of the second type is absent because

$$\begin{aligned} 2(Y_{42} + 3Y_{432}) - (Y_{42} + 1 + 3(Y_{432} + 1)) - (Y_{42} - 1 + 3(Y_{432} - 1)) = \\ = (Y_{42} + 3Y_{432})(2 - 1 - 1) + 4 - 4 = 0 \end{aligned} \quad (9)$$

The anomaly of the third type is absent because

$$\begin{aligned} 6Y_{432} - (3(Y_{432} + 1)) - (3(Y_{432} - 1)) = \\ = 3Y_{432}(2 - 1 - 1) + 3 - 3 = 0 \end{aligned} \quad (10)$$

The anomaly of the fourth type is absent if

$$Y_{42} + 3Y_{432} = 0 \quad (11)$$

The anomaly of the fifth type is absent if

$$\begin{aligned} 2(Y_{42}^3 + 3Y_{432}^3) - ((Y_{42} + 1)^3 + 3(Y_{432} + 1)^3) - ((Y_{42} - 1)^3 + 3(Y_{432} - 1)^3) = \\ = 2(Y_{42}^3 + 3Y_{432}^3) - ((Y_{42} + 1 + Y_{42} - 1)^3 - 3(Y_{42} + 1)^2(Y_{42} - 1) \\ - 3(Y_{42} - 1)^2(Y_{42} + 1)) - 3((Y_{432} + 1 + Y_{432} - 1)^3 \\ - 3(Y_{432} + 1)^2(Y_{432} - 1) - 3(Y_{432} - 1)^2(Y_{432} + 1)) \\ = 2(Y_{42}^3 + 3Y_{432}^3) - (2Y_{42}^3 + 6Y_{42}) - 3(2Y_{432}^3 + 6Y_{432}) \\ = (Y_{42}^3 + 3Y_{432}^3)(2 - 2) - 6(Y_{42} + 3Y_{432}) = 0 \end{aligned} \quad (12)$$

This means that the sum of the hypercharge over the left - handed doublets should be equal to zero:

$$Y_{42} + 3Y_{432} = 0 \quad (13)$$

3. The model should be invariant under the continuation of the Z_6 symmetry of the Standard Model. Therefore, we come to the following equations:

$$\begin{aligned} (\frac{2N}{4} + \frac{2N}{3} + N + Y_{432}N) \bmod 2 &= 0 \\ (\frac{2N}{4} + N + Y_{42}N) \bmod 2 &= 0 \\ Y_{42} + 3Y_{432} &= 0 \end{aligned} \quad (14)$$

As a result the hypercharge assignment is the following [18]. In the 5 - th row there are two elements with $Y_4 = \frac{1}{2} - 6K + 1$ and $Y'_4 = \frac{1}{2} - 6K - 1$ (were K is an arbitrary integer number). In the 6 -th row we have the only

element with $Y_{42} = \frac{1}{2} - 6K$, where K is the same as in the previous row. In the 7 - th row there are two elements with $Y_{43} = -\frac{\frac{1}{2}-6K}{3} + 1$ and $Y'_{43} = -\frac{\frac{1}{2}-6K}{3} - 1$. The 8 -th row contains the only element with $Y_{432} = -\frac{\frac{1}{2}-6K}{3}$. Again, in these two rows K is the same as before.

For the definiteness let us list here the fermions for the choice $K = 0$.

$$\begin{aligned}
U(1) : & \quad \Psi_0, \Psi_{-2}; \\
U(1), SU(2) : & \quad \Theta_{-1}^{i_2}; \\
U(1), SU(3) : & \quad \Psi_{\frac{4}{3}}^{i_3}, \Psi_{-\frac{2}{3}}^{i_3}; \\
U(1), SU(2), SU(3) : & \quad \Theta_{\frac{1}{3}}^{i_3 i_2}; \\
U(1), SU(4) : & \quad \Psi_{\frac{3}{2}}^{i_4}, \Psi_{-\frac{1}{2}}^{i_4}; \\
U(1), SU(2), SU(4) : & \quad \Theta_{\frac{1}{2}}^{i_4 i_2}; \\
U(1), SU(3), SU(4) : & \quad \Psi_{\frac{5}{6}}^{i_4 i_3}, \Psi_{-\frac{7}{6}}^{i_4 i_3}; \\
U(1), SU(2), SU(3), SU(4) : & \quad \Theta_{-\frac{1}{6}}^{i_4 i_3 i_2}; \\
& \quad \dots
\end{aligned} \tag{15}$$

In the list (15) we have specified the Standard Model fermions and the Farhi - Susskind model fermions. If the sequence (1) is restricted by these models only, the gauge group of the theory would be

$$SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)/Z_{12} \tag{16}$$

The correspondence between our notations and the conventional ones is the following (we consider the first generation only):

$$\begin{aligned}
\Psi_0 &= \nu_R; \Psi_{-2} = e_R^-; \mathcal{P}\Psi_0(t, \bar{r}) = i\nu_L(t, -\bar{r}); \mathcal{P}\Psi_{-2} = ie_L^-; \\
\Psi_{\frac{4}{3}}^{i_3} &= u_R; \Psi_{-\frac{2}{3}}^{i_3} = d_R; \mathcal{P}\Psi_{\frac{4}{3}}^{i_3} = iu_L; \mathcal{P}\Psi_{-\frac{2}{3}}^{i_3} = id_L; \\
\Psi_{\frac{3}{2}}^{i_4} &= N_R; \Psi_{-\frac{1}{2}}^{i_4} = E_R; \mathcal{P}\Psi_{\frac{3}{2}}^{i_4} = iN_L; \mathcal{P}\Psi_{-\frac{1}{2}}^{i_4} = iE_L; \\
\Psi_{\frac{5}{6}}^{i_4 i_3} &= U_R; \Psi_{-\frac{7}{6}}^{i_4 i_3} = D_R; \mathcal{P}\Psi_{\frac{5}{6}}^{i_4 i_3} = iU_L; \mathcal{P}\Psi_{-\frac{7}{6}}^{i_4 i_3} = iD_L.
\end{aligned} \tag{17}$$

It is worth mentioning that the fermions of the first generation listed here do not diagonalize the mass matrix (see discussion of the fermion masses below). Instead the certain linear combinations of the listed fermions diagonalize the mass matrix thus giving rise to mixing angles and flavor changing amplitudes.

VI. FERMION MASSES

In our construction we suppose that the formation of fermion masses is not related to the chiral symmetry breaking due to the $SU(4)$ interactions. One may suppose, for example, that the fermion masses appear at the energies much higher than the energies at which the Hypercolor tower works. Let us suppose that massless fermion is flying through a gas of objects such that inside them the transition between the states related by parity conjugation may occur. In particular, processes like that may happen within the objects, such that their interior is organized in an unusual way. Namely, suppose, that inside that objects the transformation that is seen from outside as a space inversion may happen continuously. These objects, in turn, may have an origin of gravitational nature. Probably, objects like that may belong to a class of black holes supplemented by quantum effects.

Let the density of such objects be of the order of Λ_h^3 while their size is about M_g^{-1} . Let the amplitude of the transition $\Psi \rightarrow \mathcal{P}\Psi$ be proportional to the dimensionless constant β . Then it can be easily calculated that the massless fermion becomes massive with the mass of the order of $m_\Psi \sim \beta \frac{\Lambda_h^3}{M_g^2}$. The process like this happens in the Extended Technicolor theory, where massless quark or lepton is flying through a gas of techniquarks. The ETC interactions between them and the SM fermions occur at the distances $\sim \frac{1}{M_{ETC}}$ while the density of technifermions that are condensed in vacuum is of the order of Λ_{TC} . So, the SM fermion masses are proportional to $\frac{\Lambda_{TC}^3}{M_{ETC}^2}$. However, in our consideration we suppose that the given mechanism happens due to the physics at the scales Λ_h that may

be extremely large. Λ_h may even be of the order of Plank mass. We do not require existence of the processes like ETC transition between quarks and techniquarks. Therefore, our Hypercolor model does not suffer from the problems specific for ETC models.

In order to incorporate the fermion masses to the theory we simply introduce the mass term in the action in the following way. Let us denote the right - handed fermions from the first column of (2) as $U_A = U_a^\alpha = \Psi_{Y_A+1}^A$, where α is the collection of indices of the subgroups of (1) while a enumerates generations. The pair (α, a) that identifies the fermion is denoted by A . We denote the right - handed fermions from the second column of (2) as $D_A = D_a^\alpha = \Psi_{Y_A-1}^A$. The left handed doublets are denoted $L_{Ai} = L_{ai}^\alpha = \Theta_{Y_A}^{Ai}$. The hypercharge of the left - handed fermion A is denoted by Y_A . In order to provide invariance of the mass term under the Electromagnetic $U(1)$ the correspondent right - handed fermions have hypercharges $Y_A \pm 1$. The mass term is

$$\begin{aligned} \mathcal{M} &= i \sum_U M_{ab}^U [U_b^\alpha(t, \bar{r})]^\dagger \mathcal{P} U_a^\alpha(t, -\bar{r}) + i \sum_D M_{ab}^D [D_b^\alpha(t, \bar{r})]^\dagger \mathcal{P} D_a^\alpha(t, -\bar{r}) + c.c. \\ &= \sum_U M_{ab}^U [U_b^\alpha]^\dagger \Omega_i^1 L_{ai}^\alpha + \sum_D M_{ab}^D [D_b^\alpha]^\dagger \Omega_i^2 L_{ai}^\alpha + c.c. \end{aligned} \quad (18)$$

Here the sum is over the rows of (2) to which U and D belong. The sum over a, b , and α is also implied. Before diagonalization the mass matrices M^U and M^D have block - diagonal forms. Each block corresponds to a certain collection of Hypercolor subgroups of (1) that act on the correspondent fermion states. Both M^U and M^D can always be made diagonal (with real elements) via $U(\mathcal{N})_L \otimes U(\mathcal{N})_R$ rotations together with the suitable redefinition of the θ - parameters in the $SU(N)$ theta terms of the action: $M^U \rightarrow [\mathcal{K}_L^U]^\dagger [M^U] \mathcal{K}_R^U$; $M^D \rightarrow [\mathcal{K}_L^D]^\dagger [M^D] \mathcal{K}_R^D$, where $\mathcal{K}_{L,R}^U \in U(\mathcal{N})$, $\mathcal{K}_{L,R}^D \in U(\mathcal{N})$. The dynamical part of the fermion action is invariant under these transformations if $\mathcal{K}_L^U = \mathcal{K}_L^D$. That's why in the charged weak currents the mixing matrix $[\mathcal{K}_L^U]^\dagger \mathcal{K}_L^D$ appears that contains the usual CKM matrix of the Standard Model.

Starting from the theory with diagonal real mass matrix using $U(\mathcal{N})$ transformations we can always bring the theory to the form, in which mixing is absent while the mass matrix is not diagonal but Hermitian. Namely, we can use the transformation

$$M^U \rightarrow \mathcal{K}_L^U [M^U] [\mathcal{K}_L^U]^\dagger; \quad M^D \rightarrow \mathcal{K}_L^D [M^D] [\mathcal{K}_L^D]^\dagger, \quad (19)$$

where \mathcal{K}_L^U , and \mathcal{K}_L^D are the same as above. Again M^U and M^D have block - diagonal forms. The hermitian nature of the mass matrix means, in particular, that the determinant of each mentioned block is real. That's why (19) is not accompanied by the shift in θ - parameter of $SU(N)$ theta - term for any N . Below we always imply that matrices M^U and M^D are Hermitian.

It is well - known that in such form of the theory the θ parameter for $SU(3)$ is negligible. The understanding of the reason why it is so is the subject of the so - called strong CP problem of QCD. In our model we also imply that the θ parameters for $SU(N)$ subgroups of (1) vanish while the mass matrix is Hermitian. In this language the analogue of the strong CP problem of QCD is: Why is the mass matrix Hermitian while the theta - terms are absent.

Actually, we do not try to give the answer to this question at the present moment. We can only say, that if our supposition is correct and the mass matrix is indeed Hermitian (while the theta - terms are absent), its hermitian nature must originate from the energies at which (as described above) the transition between the left - handed and the right - handed fermions occurs. Suppose that the amplitude of such process for the transformation of the fermion ψ_L to the fermion ϕ_R is $A_{\psi_L \rightarrow \phi_R}$. On this language the hermitian nature of the mass matrix means that the amplitude of the process $\psi_R \rightarrow \phi_L$ is the same: $A_{\psi_R \rightarrow \phi_L} = A_{\psi_L \rightarrow \phi_R}$. At the same time due to (18) the amplitude of the inverse process is given by the complex conjugate value: $A_{\phi_R \rightarrow \psi_L} = A_{\phi_L \rightarrow \psi_R} = A_{\psi_L \rightarrow \phi_R}^* = A_{\psi_R \rightarrow \phi_L}^*$.

The mass matrix as described by (18) does not contain Majorana mass term for the neutral right - handed neutrinos. In principle, this term can be added to the model without causing any problems. It should have the form $M^{\nu R}_{ab} \epsilon_{\alpha\beta} \nu_{R,a}^\alpha \nu_{R,b}^\beta$, where a and b indicate generations. Its high energy origin should differ from that of the Dirac mass term. Namely, Majorana mass term is related to the transformation of the right - handed neutrinos to their antiparticles. (As described above the Dirac mass term is related to the transformation of the fermion to its parity partner.) Further we do not focus on this question and imply that the model can be updated if necessary in order to provide neutrino masses in a proper way.

VII. THE SPONTANEOUS BREAKDOWN OF CHIRAL SYMMETRY

Let us suppose here that $SU(4)$ group is confining and gives rise to chiral symmetry breaking. (Later we shall discuss the conditions under which this happens.) Then the vacuum alignment [21] works in such a way that the chiral condensates must be proportional to the only explicit $SU(2)$ variable Ω .

Let us define the field Φ as follows

$$\Phi_{i_2 B}^{1A} = [\Theta_{Y_A}^{A i_2}]^+ \Psi_{Y_B+1}^B; \quad \Phi_{i_2 B}^{2A} = [\Theta_{Y_A}^{A i_2}]^+ \Psi_{Y_B-1}^B \quad (20)$$

where A, B enumerate left handed fermions (Y_A, Y_B are their hypercharges). Right handed fermions $\Psi_{Y_B+1}^B$ belong to the first column of (2) while $\Psi_{Y_B-1}^B$ belong to the second column. In the previous section we defined index A as a pair (α, a) , where α is the collection of $SU(K)$ indices ($SU(K)$ is a subgroup of (1)) while a enumerates generations. In this section the $SU(4)$ indices are ignored in this collection as we describe the effective theory, which appears after Technicolor gauge field is integrated out. We imply the mass matrix is diagonal in $SU(4)$ index. In (20) summation over $SU(4)$ index is implied. Below we omit indices A and B and imply that Φ_j^i is $\mathcal{N} \times \mathcal{N}$ matrix for each i and j . Then the mass term in the action can be written as

$$\mathcal{M} = \text{Tr}[\Phi_j^i]^+ \mathcal{M}_j^i + c.c. \quad (21)$$

Here the mass matrix is $\mathcal{M}_j^i = [\mathcal{M}_j^i]_B^A = [\mathcal{M}_j^i]_{b\alpha}^{a\alpha}$. It is expressed through M_{ab}^U and M_{ab}^D as follows:

$$\begin{aligned} [\mathcal{M}_i^1]_{b\beta}^{a\alpha} &= M_{ab}^U \Omega_i^1 \delta_{\alpha\beta} \\ [\mathcal{M}_i^2]_{b\beta}^{a\alpha} &= M_{ab}^D \Omega_i^2 \delta_{\alpha\beta} \end{aligned} \quad (22)$$

If all interactions but the Technicolor and the fermion masses are switched off, then the Technicolor theory has the symmetry $SU(2\mathcal{N})_L \otimes SU(2\mathcal{N})_R \otimes U(1)_V$, where \mathcal{N} is the whole number of the left handed doublets ($A, B = 1, \dots, \mathcal{N}$). $U(1)_V$ acts identically on left - handed and right handed fermions. $(U(1)_A)$ is not a quantum symmetry due to the anomaly.) The effective action is

$$S(\Phi) = c_1 \text{Tr} [\mathcal{D}\Phi]^+ \mathcal{D}\Phi + V(\Phi) \quad (23)$$

where the potential $V(\Phi)$ has the form

$$\begin{aligned} V(\Phi) &= c_2 (\text{Tr} [\Phi_j^i]^+ \Phi_j^i - \kappa^2)^2 + c_3 \text{Tr} [\Phi_{j_1}^{i_1}]^+ \Phi_{j_2}^{i_1} [\Phi_{j_2}^{i_2}]^+ \Phi_{j_1}^{i_2} \\ &\quad - \text{Tr} [\Phi_j^i]^+ \mathcal{M}_j^i - \text{Tr} \Phi_j^i [\mathcal{M}_j^i]^+ \end{aligned} \quad (24)$$

In the above expressions κ , and c_k are unknown constants. The derivative \mathcal{D} contains all gauge fields but the Technicolor field. \mathcal{M} is the mass matrix. The terms with higher derivatives and higher powers of Φ (for example, those that contain terms with the determinant) are not relevant at low enough energies.

The physical processes described by (23) happen within the regions of space with the typical size $\frac{1}{\Lambda_{TC}}$, where Λ_{TC} is the analogue of Λ_{QCD} . The duration of such processes can also be estimated as $\frac{1}{\Lambda_{TC}}$. Therefore the estimate of the mass term fluctuation for the technifermion with mass m_T is $\sim m_T \frac{1}{\Lambda_{TC}} \delta\Phi$. In case when the technifermion mass m_T is much larger than Λ_{TC} this term dominates and the fluctuations of the field Φ composed of this technifermion can be estimated as $\delta\Phi \sim \frac{\Lambda_{TC}^4}{m_T}$. The energy ϵ in the processes to be described by (23) must be much less than the scale of $\delta\Phi$. This is needed in order to neglect nonlocal terms with higher powers of the derivatives. That's why, (23) may be relevant for $\frac{\epsilon}{\Lambda_{TC}} \ll [\frac{\Lambda_{TC}}{m_T}]^{1/3}$. In particular, for $m_T \sim 1000$ Tev, $\Lambda_{TC} \sim 1$ Tev we would have $\epsilon \ll 100$ Gev while for $m_T \sim 10$ Tev one should require $\epsilon \ll 300$ Gev. At the energies of the order of the Electroweak scale ~ 100 Gev such extra massive technifermions must not enter the effective action in the form (23). Actually, the correspondent bilinear forms Φ defined in (20) are suppressed by the factors $\frac{m_T}{\Lambda_{TC}}$. That's why effective action (23) contains the field Φ composed of only those technifermions, the masses of which is of the order of 1 Tev and smaller. Therefore, fluctuations of Φ can be estimated as $\delta\Phi \sim \Lambda_{TC}^3$. Thus higher derivatives in (23) are suppressed at the energies $\epsilon \ll \Lambda_{TC}$. Under the same condition one can also neglect higher powers of Φ in (24). (The terms with the powers of Φ up to the fourth are needed in order to provide nonzero vacuum average of Φ .)

It is worth mentioning that (24) is invariant under the chiral $U(1)_A$ if $\mathcal{M} = 0$. This is relevant at $\epsilon \ll \Lambda_{TC}$ only. At $\epsilon \sim \Lambda_{TC}$ the terms should appear that violate $U(1)_A$ (in particular, the term that contains $\det \Phi$). The situation here differs from that of the QCD with two or three light quarks. There the effective potential contains the determinant of Φ already at the energies much less than Λ_{QCD} as the determinant contains the second and the third powers of Φ respectively. In our case the expected number of technifermions with masses around 1 Tev is larger than 4. That's why the determinant contains higher powers of Φ .

The first term in the effective action gives masses for W and Z bosons. The next terms resolve the vacuum alignment problem. The true vacuum corresponds to the minimum of the potential $V(\Phi)$. (We neglect here the perturbations due to the Standard Model interactions and the $SU(K)$ Hypercolor interactions for $K > 4$.)

In order to demonstrate how the vacuum alignment works let us consider first the simplified situation when $\mathcal{N} = 1$, $\mathcal{M}_i^j = m_j \Omega_i^j$ (values m_1 and m_2 are eigenvalues of \mathcal{M} ; no sum over j is implied in the definition of \mathcal{M}).

Let the effective potential for the field Φ has the simplified form with $c_3 = 0$:

$$V(\Phi) = c_2(\text{Tr } \Phi^\dagger \Phi - \kappa^2)^2 - [\Phi_j^i]^* \mathcal{M}_j^i - \Phi_j^i [\mathcal{M}_j^i]^* \quad (25)$$

It is clear that the vacuum value of Φ is proportional to \mathcal{M} . Thus

$$\Phi_{vac} = f \mathcal{M}, \quad (26)$$

where f is the solution of the equation:

$$0 = 2c_2(f^2(m_1^2 + m_2^2) - \kappa^2)f - 1 \quad (27)$$

In particular, if $\sqrt{m_1^2 + m_2^2} \ll 4c_2\kappa^3$, then $\Phi_{vac} = (\frac{\kappa}{\sqrt{m_1^2 + m_2^2}} + \frac{1}{4c_2\kappa^2} + O(\frac{\sqrt{m_1^2 + m_2^2}}{4c_2\kappa^3}))\mathcal{M}$. Rough estimates show, however, that this limiting case is not too realistic. Λ_{TC} is to be of the order of 1 TeV. This is due to the estimate of the weak gauge boson masses: $M_Z, M_W \sim g_W \sqrt{c_2} \Phi_{vac} \sim 0.1 \frac{\Phi_{vac}}{\Lambda_{TC}}$. In case $\sqrt{m_1^2 + m_2^2} \ll 4c_2\kappa^3 \sim \Lambda_{TC}$ we have $M_Z, M_W \sim 0.1\Lambda_{TC}$. So, in this case the technifermion masses are somewhere around 100 GeV, or even less. This situation cannot be considered as realistic. That's why we expect that the masses of the technifermions are to be of the order of $\Lambda_{TC} \sim 1$ TeV.

For any relation between technifermion masses and Λ_{TC} due to (26) we have

$$\begin{aligned} \langle [\Theta_Y^{i_4 i_2}]^+ \Psi_{Y+1}^{i_4} \rangle &\sim m_1 \Omega_{i_2}^1 \\ \langle [\Theta_Y^{i_4 i_2}]^+ \Psi_{Y-1}^{i_4} \rangle &\sim m_2 \Omega_{i_2}^2 \end{aligned} \quad (28)$$

Now we come back to the general case of $\mathcal{N} \neq 1$ and the effective action (24). Minimum of the effective potential is achieved at the vacuum value Φ_{vac} . Let us introduce the $SU(2\mathcal{N})$ index $\mathbf{a} = (A, i)$. Both $\mathcal{M}_{\mathbf{b}}^{\mathbf{a}}$ and $[\Phi_{vac}]_{\mathbf{b}}^{\mathbf{a}}$ are $2\mathcal{N} \times 2\mathcal{N}$ matrices. Φ_{vac} satisfies the equation

$$\mathcal{M} = 2c_2 \Phi \text{Tr } [\Phi]^\dagger \Phi - 2c_2 \kappa^2 \Phi + 2c_3 \Phi [\Phi]^\dagger \Phi \quad (29)$$

Using $SU(2\mathcal{N})_{L,R}$ rotations we can always make the mass matrix diagonal. Let us denote its diagonal elements m_i . It is easy to understand, that the matrix Φ_{vac} also becomes diagonal. We denote its diagonal elements ϕ_i . Thus (29) leads to

$$m_i = 2c_2 \phi_i \sum |\phi_i|^2 - 2c_2 \kappa^2 \phi_i + 2c_3 \phi_i |\phi_i|^2 \quad (30)$$

In particular, in case $\frac{m_i(2\mathcal{N} + \frac{c_3}{c_2})^{\frac{3}{2}}}{4\kappa^3 c_3} \ll 1$ we have

$$\phi_i = \frac{\kappa}{\sqrt{2\mathcal{N} + \frac{c_3}{c_2}}} + \frac{1}{4\kappa^2 c_3} ((2\mathcal{N} + \frac{c_3}{c_2})m_i - \sum m_i) + O([\frac{m_i(2\mathcal{N} + \frac{c_3}{c_2})^{\frac{3}{2}}}{\kappa^3 c_3}]^2) \quad (31)$$

(However, as for the simplified model considered above, this case is not realistic and we actually have to consider technifermion masses to be of the order of 1 TeV.)

For any values of m_i let us consider the vacuum value of Φ written in the form $[(\Phi_{vac})_j^i]_{b\beta}^{a\alpha}$, where α, β denote the collection of $\dots SU(6) \otimes SU(5) \otimes SU(3)$ indices while a, b enumerate generations. Symmetry properties of Φ_{vac} are obvious. In Unitary gauge $\Omega = \mathbf{1}$ the mass matrix is such that $\mathcal{M}_1^2 = \mathcal{M}_2^1 = 0$. In this case $(\Phi_{vac})_2^1 = (\Phi_{vac})_1^2 = 0$. One can easily see, that Φ_{vac} preserves all symmetries of \mathcal{M} . Namely, let us rewrite the mass matrix in the form $[\mathcal{M}_j^i]_{b\beta}^{a\alpha}$. Then $[\mathcal{M}_j^i]_{b\beta}^{a\alpha}$ is nonzero only if α is identical to β . Therefore, $[(\Phi_{vac})_j^i]_{b\beta}^{a\alpha} \neq 0$ only if α coincides with β . That's why Technicolor breaks the Electroweak symmetry only.

In Unitary gauge the fields of W and Z bosons as well as the Electromagnetic field A are defined as usual. The mass matrix and $[\Phi_{vac}]_j^i$ are invariant under the Electromagnetic $U(1)$ symmetry. At the same time $[\Phi_{vac}]_j^i$ breaks Electroweak $SU(2)$ and the Hypercharge $U(1)$. Therefore, the W and Z bosons acquire their masses while A remains massless.

We suppose that coefficients c_2 and c_3 are real while matrix \mathcal{M} is Hermitian. That's why $[\Phi_{vac}]_j^i$ is Hermitian. We can define the four - component spinors $u_A = \begin{pmatrix} \Psi_{Y_A}^A \\ \Theta_{Y_A}^{A\dagger 1} \end{pmatrix}$ and $d_A = \begin{pmatrix} \Psi_{Y_A}^A \\ \Theta_{Y_A}^{A2} \end{pmatrix}$. Then the technipion condensate vanishes:

$$\begin{aligned} \langle \bar{u}_A \gamma_5 u_B \rangle &= \langle [\Theta_{Y_A}^{A1}]^+ \Psi_{Y_A+1}^A - [\Psi_{Y_A+1}^A]^+ \Theta_{Y_A}^{A1} \rangle = 0 \\ \langle \bar{d}_A \gamma_5 d_B \rangle &= \langle [\Theta_{Y_A}^{A2}]^+ \Psi_{Y_A-1}^A - [\Psi_{Y_A-1}^A]^+ \Theta_{Y_A}^{A2} \rangle = 0 \end{aligned} \quad (32)$$

The physical sense of (32) is trivial. It means that the Technicolor vacuum is invariant under the space inversion.

The next step in the consideration of the vacuum alignment would be to take into account small perturbations due to the Standard Model interactions (and due to the other interactions corresponding to the subgroups of (1)). It was found in [21] that due to the Standard Model interactions the conventional form of the chiral condensate appears. We suppose, that the higher subgroups of (1) do not introduce anything new. Up to this assumption we come to the conclusion that in our case Technicolor breaks the Electroweak symmetry properly.

VIII. THE FURTHER CONTINUATION

The next step of our investigation is the analysis of the sequence (15). Let us notice that the second two rows are actually the copy of the first two rows supplemented by an additional $SU(3)$ index. Next, the second four rows are again the copy of the first four rows supplemented by an additional $SU(4)$ index. Let us suppose that this process is repeated infinitely. Then, the next 8 rows in the sequence are added in the form:

$$\begin{aligned} & \dots \\ U(1), SU(5) : & \quad \Psi_{Y_5}^{i_5}, \Psi_{Y_5'}^{i_5}; \\ U(1), SU(2), SU(5) : & \quad \Theta_{Y_{52}}^{i_5 i_2}; \\ U(1), SU(3), SU(5) : & \quad \Psi_{Y_{53}}^{i_5 i_3}, \Psi_{Y_{53}'}^{i_5 i_3}; \\ U(1), SU(2), SU(3), SU(5) : & \quad \Theta_{Y_{532}}^{i_5 i_3 i_2}; \\ U(1), SU(4), SU(5) : & \quad \Psi_{Y_{54}}^{i_5 i_4}, \Psi_{Y_{54}'}^{i_5 i_4}; \\ U(1), SU(2), SU(4), SU(5) : & \quad \Theta_{Y_{542}}^{i_5 i_4 i_2}; \\ U(1), SU(3), SU(4), SU(5) : & \quad \Psi_{Y_{543}}^{i_5 i_4 i_3}, \Psi_{Y_{543}'}^{i_5 i_4 i_3}; \\ U(1), SU(2), SU(3), SU(4), SU(5) : & \quad \Theta_{Y_{5432}}^{i_5 i_4 i_3 i_2}; \\ & \dots \end{aligned} \quad (33)$$

Again, we choose the hypercharge assignment in such a way that:

1. Mass terms for the fermions proportional to $\Psi^+(t, \vec{r}) \mathcal{P} \Psi(t, -\vec{r})$ are invariant under Electromagnetic $U(1)$. Therefore

$$\begin{aligned} Y_5 &= Y_{52} + 1; Y_5' = Y_{52} - 1; \\ Y_{53} &= Y_{532} + 1; Y_{53}' = Y_{532} - 1; \\ Y_{54} &= Y_{542} + 1; Y_{54}' = Y_{542} - 1; \\ Y_{543} &= Y_{5432} + 1; Y_{543}' = Y_{5432} - 1 \end{aligned} \quad (34)$$

2. Chiral anomaly should vanish. The consideration is similar to that of section 5. Namely, there may appear the new anomalies of the following types:

$$\begin{aligned} & 1) SU(5) - SU(5) - SU(5) \\ & 2) SU(5) - SU(5) - U(1) \\ & 3) SU(4) - SU(4) - U(1) \\ & 4) SU(3) - SU(3) - U(1) \\ & 5) SU(2) - SU(2) - U(1) \\ & 6) U(1) - U(1) - U(1) \end{aligned} \quad (35)$$

The anomaly of the first type vanishes because the number of left - handed fermions is equal to the number of the right - handed ones while both types of fermions belong to the fundamental representation of $SU(5)$.

The anomalies of the second, the third, and the fourth types vanish due to (34).

The anomalies of the fifth and the sixth types vanish if the sum of the hypercharge over left - handed doublets is zero. Thus

$$Y_{52} + 3Y_{532} + 4Y_{542} + 4 \times 3 \times Y_{5432} = 0 \quad (36)$$

3. The model must be invariant under the further continuation of the Z_{12} symmetry of (16). Therefore

$$\begin{aligned} & \left(\frac{2N}{5} + \frac{2N}{4} + \frac{2N}{3} + N + Y_{5432}N \right) \bmod 2 = 0 \\ & \left(\frac{2N}{5} + \frac{2N}{4} + N + Y_{542}N \right) \bmod 2 = 0 \\ & \left(\frac{2N}{5} + \frac{2N}{3} + N + Y_{532}N \right) \bmod 2 = 0 \\ & \left(\frac{2N}{5} + N + Y_{52}N \right) \bmod 2 = 0 \\ & Y_{52} + 3Y_{532} + 4Y_{542} + 4 \times 3 \times Y_{5432} = 0 \end{aligned} \quad (37)$$

The solution is

$$\begin{aligned} Y_{52} &= \frac{3}{5} - 2(3K_{532} + 4K_{542} + 12K_{5432}); Y_5 = Y_{52} + 1; Y'_5 = Y_{52} - 1; \\ Y_{532} &= \frac{29}{15} + 2K_{532}; Y_{53} = \frac{44}{15} + 2K_{532}; Y'_{53} = \frac{14}{15} + 2K_{532}; \\ Y_{542} &= \frac{1}{10} + 2K_{542}; Y_{54} = \frac{11}{10} + 2K_{542}; Y'_{54} = -\frac{9}{10} + 2K_{542}; \\ Y_{5432} &= -\frac{17}{30} + 2K_{5432}; Y_{543} = \frac{13}{30} + 2K_{5432}; Y'_{543} = -\frac{47}{30} + 2K_{5432} \end{aligned} \quad (38)$$

Here $K_{532}, K_{542}, K_{532}$ are arbitrary integer numbers.

IX. HIGHER HYPERCOLOR GROUPS

In this section we derive the hypercharge assignment for all fermions of our model. We require that the chiral anomaly is absent and the additional \mathcal{Z} symmetry is preserved. Actually, the fact that there exists such a solution is nontrivial. A priori it is not clear that it is possible to satisfy both requirements simultaneously.

Let us continue the sequence (33) infinitely. It has the form:

$$\begin{aligned} & \dots \\ & U(1), SU(5) : \quad \Psi_{Y_5}^{i_5}, \Psi_{Y'_5}^{i_5}; \\ & U(1), SU(2), SU(5) : \quad \Theta_{Y_{52}}^{i_5 i_2}; \\ & U(1), SU(3), SU(5) : \quad \Psi_{Y_{53}}^{i_5 i_3}, \Psi_{Y'_{53}}^{i_5 i_3}; \\ & U(1), SU(2), SU(3), SU(5) : \quad \Theta_{Y_{532}}^{i_5 i_3 i_2}; \\ & U(1), SU(4), SU(5) : \quad \Psi_{Y_{54}}^{i_5 i_4}, \Psi_{Y'_{54}}^{i_5 i_4}; \\ & U(1), SU(2), SU(4), SU(5) : \quad \Theta_{Y_{542}}^{i_5 i_4 i_2}; \\ & U(1), SU(3), SU(4), SU(5) : \quad \Psi_{Y_{543}}^{i_5 i_4 i_3}, \Psi_{Y'_{543}}^{i_5 i_4 i_3}; \\ & U(1), SU(2), SU(3), SU(4), SU(5) : \quad \Theta_{Y_{5432}}^{i_5 i_4 i_3 i_2}; \\ & \dots \\ & U(1), \dots, SU(K) : \quad \Psi_{Y_{K\dots}}^{i_K \dots}, \Psi_{Y'_{K\dots}}^{i_K \dots}; \\ & U(1), SU(2), \dots, SU(K) : \quad \Theta_{Y_{K\dots 2}}^{i_K \dots i_2}; \\ & \dots \end{aligned} \quad (39)$$

Now we require that the chiral anomaly is absent while the gauge group is (1), where \mathcal{Z} is defined by (6). Below we prove that **the necessary hypercharge assignment is**

$$\begin{aligned} Y_2 &= -1 \\ Y_{i_1 i_2 i_3 \dots i_{M-1} i_M 2} &= -1 + 2\left(1 - \frac{1}{i_M}\right) + 2 \sum_{k=1}^{M-1} \left[\theta(i_k - i_{k+1} - 1) - \frac{1}{i_k}\right] + 2N_{i_1 i_2 i_3 \dots i_{M-1} i_M 2} \\ Y_{ij\dots l} &= Y_{ij\dots l2} + 1; Y'_{ij\dots l} = Y_{ij\dots l2} - 1 \end{aligned} \quad (40)$$

where $\theta(x) = 1$ for $x > 0$; $\theta(x) = 0$ for $x \leq 0$. In the second row $M \geq 1$. For any K integer numbers $N_{i_1 i_2 i_3 \dots i_{M-1} i_M 2}$ entering (40) must satisfy the equation

$$\sum_{K > i > j > \dots > l > 2} ij\dots l N_{Kij\dots l2} = 0 \quad (41)$$

Here the sum is over any (unordered) sets of different integer numbers i, j, \dots, l such that $2 < i, j, \dots, l < K$.

The proof is as follows. First of all, if (6) is the symmetry of the theory then the recursion relations take place:

$$Y_{Kij\dots l2} = Y_{ij\dots l2} - \frac{2}{K} + 2M_{Kij\dots l2}; Y_{Kij\dots l} = Y_{Kij\dots l2} + 1; Y'_{Kij\dots l} = Y_{Kij\dots l2} - 1, \quad (42)$$

where $M_{Kij\dots l2}$ is an integer number.

Let us require that for any K

$$\sum_{K > i > j > \dots > l > 2} ij\dots l Y_{Kij\dots l2} = 0, \quad (43)$$

This means that the chiral anomaly is absent even if the sequence (1) is ended at the $SU(K)$ factor with any value of K .

Namely. there may appear the new anomalies of the following types:

$$\begin{aligned} 1) & SU(N) - SU(N) - SU(N), N > 2 \\ 2) & SU(N) - SU(N) - U(1), N > 2 \\ 3) & SU(2) - SU(2) - U(1) \\ 4) & U(1) - U(1) - U(1) \end{aligned} \quad (44)$$

The anomaly of the first type vanishes because the number of left - handed fermions is equal to the number of the right - handed ones while both types of fermions belong to the fundamental representation of $SU(N)$.

The anomalies of the second type vanish because $Y_{ij\dots l} = Y_{ij\dots l2} + 1$; $Y'_{ij\dots l} = Y_{ij\dots l2} - 1$.

The anomalies of the third and the fourth types vanish if the sum of the hypercharge over left - handed doublets is zero. This leads to (43).

Below we prove that for any K integer numbers $M_{Kij\dots l2}$ can be chosen in such a way, that (43) is satisfied. Let $\sum_{K' > i > j > \dots > l > 2} ij\dots l Y_{K'ij\dots l2} = 0$ for $K' < K$ (this was demonstrated already for $K' = 4$). Then

$$\begin{aligned} \sum_{K > i > j > \dots > l > 2} ij\dots l Y_{Kij\dots l2} &= \sum_{K > i > j > \dots > l > 2} ij\dots l Y_{ij\dots l2} \\ &- \frac{2}{K} \sum_{K > i > j > \dots > l > 2} ij\dots l + 2 \sum_{K > i > j > \dots > l > 2} ij\dots l M_{Kij\dots l2} \\ &= -\frac{2}{K} \sum_{K > i > j > \dots > l > 2} ij\dots l + 2 \sum_{K > i > j > \dots > l > 2} ij\dots l M_{Kij\dots l2} \\ &= -2 \frac{K!}{3!K} + 2 \sum_{K > i > j > \dots > l > 2} ij\dots l M_{Kij\dots l2} \end{aligned} \quad (45)$$

Here we used the identity

$$\sum_{K > i > j > \dots > l > 2} ij\dots l = \frac{K!}{3!} \quad (46)$$

The derivation of (46) is as follows. Suppose that (46) is valid for a certain number of K (this is evident, for example, for $K = 4$). Then

$$\begin{aligned}
\sum_{2 < ij \dots l < K+1} ij \dots l &= 1 + 3 \frac{3!}{3!} + 4 \frac{4!}{3!} + 5 \frac{5!}{3!} + \dots + K \frac{K!}{3!} \\
&= \frac{1}{3!} (3! + 3!3 + 4!4 + \dots + K!K) \\
&= \frac{1}{3!} (4! + 4!4 + \dots + K!K) \\
&= \frac{1}{3!} (K+1)!
\end{aligned} \tag{47}$$

From (45) it is clear that for $K > 3$ it is always possible to choose one of the values $M_{Kij \dots l2}$ in such a way that (43) is satisfied. The same statement for $K = 3$ is also valid as follows from the consideration of the Standard Model.

Let us now introduce the following notations:

$$\begin{aligned}
M_{Kij \dots l2} &= M'_{Kij \dots l2} + 1, \text{ for } K-1 > i > j > \dots > l > 2; \\
M_{Kij \dots l2} &= M'_{Kij \dots l2}, \text{ for } K-1 = i > j > \dots > l > 2
\end{aligned} \tag{48}$$

Then

$$-\frac{K!}{3!K} + \sum_{K > i > j > \dots > l > 2} ij \dots l M_{Kij \dots l2} = \sum_{K > i > j > \dots > l > 2} ij \dots l M'_{Kij \dots l2} \tag{49}$$

The relations that define the fermion hypercharges can be rewritten in the following way:

$$\begin{aligned}
Y_{Kij \dots l} &= Y_{Kij \dots l2} + 1; Y'_{Kij \dots l} = Y_{Kij \dots l2} - 1, \\
Y_{Kij \dots l2} &= Y_{ij \dots l2} - \frac{2}{K} + 2 + 2M'_{Kij \dots l2} \\
&\text{(for } K-1 > i > j > \dots > l > 2, \text{ or } K = 3); \\
Y_{Kij \dots l2} &= Y_{ij \dots l2} - \frac{2}{K} + 2M'_{Kij \dots l2} \\
&\text{(for } K-1 = i > j > \dots > l > 2)
\end{aligned} \tag{50}$$

Here integer numbers $M'_{Kij \dots l2}$ are chosen in such a way that

$$\sum_{K > i > j > \dots > l > 2} ij \dots l M'_{Kij \dots l2} = 0 \tag{51}$$

Finally we come to the solution of (43) in the form (40). In particular, the choice $N_{i_1 i_2 i_3 \dots i_{M-1} i_M 2} = 0$ corresponds to

$$Y_{i_1 i_2 i_3 \dots i_{M-1} i_M 2} = -1 + 2\left(1 - \frac{1}{i_M}\right) + 2 \sum_{k=1}^{M-1} \left[\theta(i_k - i_{k+1} - 1) - \frac{1}{i_k}\right] \tag{52}$$

Thus the additional symmetry (6) fixes the hypercharge assignment up to the choice of integer numbers $N_{i_1 i_2 i_3 \dots i_{M-1} i_M 2}$ such that (41) is satisfied. We cannot eliminate this uncertainty at this stage.

X. THE RELATION BETWEEN THE DISCRETE \mathcal{Z} SYMMETRY AND THE MONOPOLE CONTENT OF THE UNIFIED MODEL.

In the previous sections we apply the additional \mathcal{Z} symmetry to the construction of the Hypercolor model. The main reason for us to do so is that the Z_6 symmetry of the Standard Model seems to us so peculiar, that we expect it must be present in a certain form in the completion of the Standard Model. Of course, the form (6) of the continuation of this symmetry is just our supposition.

The observability of the additional \mathcal{Z} symmetry of the Hypercolor model must be related to the topological objects existing within the more fundamental theory that has our tower of Hypercolor groups as a description of the low

energy approximation. Let us consider the construction of the monopole configuration (see, for example, [17]) of the hypothetical Unified model.

We fix the closed surface Σ in 4-dimensional space R^4 . For any closed loop \mathcal{C} , which winds around this surface, we may calculate the Wilson loops $\Pi_K = \text{P exp}(i \int_{\mathcal{C}} H_K^\mu dx^\mu)$, $\Gamma = \text{P exp}(i \int_{\mathcal{C}} C^\mu dx^\mu)$, $U = \text{P exp}(i \int_{\mathcal{C}} A^\mu dx^\mu)$, and $e^{i\theta} = \text{exp}(i \int_{\mathcal{C}} B^\mu dx^\mu)$, where H_K , C , A , and B are correspondingly $SU(K)$, $SU(3)$, $SU(2)$ and $U(1)$ gauge fields of the model. In the usual realization of the Hypercolor model with the gauge group $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)$ such Wilson loops should tend to unity, when the length of \mathcal{C} tends to zero ($|\mathcal{C}| \rightarrow 0$). However, in the $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}$ gauge theory the following values of the Wilson loops are allowed at $|\mathcal{C}| \rightarrow 0$:

$$\begin{aligned}\Pi_K &= e^{N \frac{2\pi i}{K}} \\ \Gamma &= e^{N \frac{2\pi i}{3}} \\ U &= e^{N\pi i} \\ e^{i\theta} &= e^{N\pi i},\end{aligned}\tag{53}$$

where $N \in \mathbb{Z}$. Then the surface Σ may carry $SU(K)/Z_K$ flux $\pi[N \bmod K]$.

Any configuration with the singularity of the type (53) could be eliminated via a singular gauge transformation. Therefore the appearance of such configurations in the theory with the gauge group $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}$ does not influence the dynamics.

Now let us consider an open surface Σ . Let the small vicinity of its boundary $U(\partial\Sigma)$ represent a point - like soliton state of the unified theory. This means that the fields of the Hypercolor model are defined now everywhere except $U(\partial\Sigma)$. Let us consider such a configuration, that for infinitely small contours \mathcal{C} (winding around Σ) the mentioned above Wilson loops are expressed as in (53). For $N \neq 0$ it is not possible to expand the definition of such a configuration to $U(\partial\Sigma)$. However, this could become possible within the unified model if the gauge group of the Hypercolor model $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}$ is embedded into the simply connected group \mathcal{H} . This follows immediately from the fact that any closed loop in such \mathcal{H} can be deformed smoothly to a point and this point could be moved to unity. Actually, for such \mathcal{H} we have $\pi_2(\mathcal{H}/[\dots \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}]) = \pi_1(\dots \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z})$. This means that in such unified model the monopole-like soliton states are allowed. The configurations with (53) and $N \neq 0$ represent fundamental monopoles of the unified model[26]. The other monopoles could be constructed of the fundamental monopoles as of the building blocks. In the unified model, which breaks down to the Hypercolor model with the gauge group $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)$ such configurations for $N \neq 0$ are simply not allowed.

The unified model, which breaks down to the Hypercolor model with the gauge group $\dots \otimes SU(3) \otimes SU(2) \otimes U(1)$ also contains monopoles because $\pi_2(\mathcal{H}/[\dots \otimes SU(3) \otimes SU(2) \otimes U(1)]) = \pi_1(\dots \otimes SU(3) \otimes SU(2) \otimes U(1)) = \mathbb{Z}$. They correspond to the Dirac strings with $\int_{\mathcal{C}} B^\mu dx^\mu = 2Q_{max}\pi K$, $K \in \mathbb{Z}$. (We suppose here that the hypercharges of the fermions are rational numbers $\frac{P}{Q}$ with integer P and Q , and the maximal value of Q is Q_{max} .) Those monopoles should be distinguished from the monopoles of the Hypercolor model with the additional discrete symmetry via counting their hypercharge $U(1)$ magnetic flux.

Using an analogy with t'Hooft - Polyakov monopoles[22] we can estimate masses of the monopole of the hypothetical Unified theory (in the presence of \mathcal{Z} symmetry) as

$$M_N \sim 4\pi\Lambda_h \left[\frac{1}{g_{U(1)}} + \sum_K \frac{1}{g_{SU(K)}} \right] N\tag{54}$$

In this sum the term corresponding to $SU(K)$ is absent if $N/K \in \mathbb{Z}$ because in this case the monopole does not carry $SU(K)/Z_K$ flux. Here Λ_h is the Unification scale and πN is the Hypercolor flux carried by the monopole.

It is worth mentioning that the usual magnetic flux of the given monopoles is 2π . This follows simply from the expression for the Electromagnetic field through the $SU(2)$ field A and the hypercharge field B :

$$A_{em} = 2B + 2\sin^2\theta_W(A_3 - B)\tag{55}$$

The mentioned above monopoles have nontrivial $SU(2)/Z_2$ flux that cancels the hypercharge flux within the second term of (55). That's why their usual flux (with respect to the Electromagnetic $U(1)$) corresponds only to the first term in (55) and is equal to 2π .

If hypercharge flux is proportional to 2π then the monopole must not necessarily carry $SU(2)/Z_2$ flux. In this case the field A_3 does not give any contribution to the Electromagnetic flux. And the monopole may carry the usual magnetic flux proportional to $4\pi\cos^2\theta_W$ due to both terms of (55).

Let us suppose that at the Unification scale all couplings become close to each other: $g^2 = g_{U(1)}^2(\Lambda_h) \sim g_{SU(2)}^2(\Lambda_h) \sim \dots \sim g_{SU(K)}^2(\Lambda_h) \sim \dots$ (In general case this is not necessarily so. For example, in the models of the so - called Petite

Unification that occurs at a TeV scale the gauge couplings are not close to each other [23].) We also suppose that the sequence (1) is ended at the Hypercolor group $SU(K_{max})$. Then

$$M_N > \frac{4\pi\Lambda_h}{g} K_{max} \quad (56)$$

So, we come to the conclusion that in case the \mathcal{Z} symmetry is present the appearance of monopoles in a hypothetical Unified theory is highly suppressed.

Let us now consider the case, when the gauge group of the Hypercolor tower is $G = \dots \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)$ while the sequence of fermions is still given by (39). The hypercharge assignment is such that any of the subgroups of \mathcal{Z} is not a symmetry of the theory. We again suppose that the hypercharges of the fermions are rational numbers $\frac{P}{Q}$ with integer P and Q , and the maximal value of Q is Q_{max} . The hypercharge of the left-handed quarks is $\frac{1}{3}$. That's why $Q_{max} \geq 3$. The smallest possible hypercharge flux of the monopole is $2Q_{max}\pi$. The groups $SU(N)$ are not involved in such monopole configurations. The magnetic flux of the monopole is proportional to $4Q_{max}\pi \cos^2\theta_W$. The estimate of the minimal monopole mass is

$$M \sim \frac{8Q_{max}\pi\Lambda_h}{g} \quad (57)$$

In the case, when a certain subgroup of \mathcal{Z} serves as a symmetry group given by the hypercharge assignment of (39), the minimal monopole mass may be larger than (57). And if this subgroup contains Z_2 then the monopoles may appear that carry usual magnetic flux proportional to Z_2 . In order to increase the minimal monopole mass one may increase Q_{max} or to make larger and larger subgroup of \mathcal{Z} the symmetry of the theory. That's why the requirement that the whole \mathcal{Z} is the symmetry is one of the ways to suppress monopoles (although, not the only one).

For the definiteness, let us demonstrate how, in principle, the Technicolor and the Standard Model interactions may be unified in a common gauge group. Here we do not consider higher Hypercolor groups and follow the construction suggested in [18]. We do not discuss the details of the breakdown mechanism and how the chiral anomaly cancellation is provided within the given scheme of Unification. Our aim here is to demonstrate how the additional discrete symmetry (6) may appear during the breakdown of Unified gauge symmetry.

Let $SU(10)$ be the Unified gauge group. The breakdown pattern is $SU(10) \rightarrow SU(4) \otimes SU(3) \times SU(2) \times U(1)/Z_{12}$. We suppose that at low energies the $SU(10)$ parallel transporter has the form:

$$\Omega = \begin{pmatrix} \Theta e^{-\frac{2i\theta}{4}} & 0 & 0 & 0 \\ 0 & \Gamma^+ e^{\frac{2i\theta}{3}} & 0 & 0 \\ 0 & 0 & U e^{-i\theta} & 0 \\ 0 & 0 & 0 & e^{2i\theta} \end{pmatrix} \in SU(10), \quad (58)$$

The fermions of each generation $\Psi_{j_1 \dots j_K}^{i_1 \dots i_N}$ carry indices i_k of the fundamental representation of $SU(10)$ and the indices j_k of the conjugate representation. They may be identified with the Standard Model fermions and Farhi - Susskind fermions as follows (we consider here the first generation only):

$$\begin{aligned} \Psi^{10} &= e_R^c; \Psi_{10}^{10} = \nu_R; \Psi^{i_2} = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}; \\ \Psi^{i_3} &= d_{i_3,R}^c; \Psi_{10}^{i_3} = u_{i_3,R}^c; \Psi^{i_2}_{i_3} = \begin{pmatrix} u_L^{i_3} \\ d_L^{i_3} \end{pmatrix}; \\ \Psi_{i_4} &= E_{i_4,R}^c; \Psi_{10,i_4} = N_{i_4,R}^c; \Psi^{i_2}_{i_4} = \begin{pmatrix} N_L^{i_4} \\ E_L^{i_4} \end{pmatrix}; \\ \Psi^{i_3}_{i_4} &= D_{i_3 i_4,R}^c; \Psi_{10,i_4}^{i_3} = U_{i_3 i_4,R}^c; \Psi^{i_2}_{i_3 i_4} = \begin{pmatrix} U_L^{i_3 i_4} \\ D_L^{i_3 i_4} \end{pmatrix} \\ (i_2 &= 8, 9; i_3 = 5, 6, 7; i_4 = 1, 2, 3, 4); \end{aligned} \quad (59)$$

Here the charge conjugation is defined as follows: $f_\alpha^c = \epsilon_{\alpha\beta} [f^\beta]^*$.

In principle the fermion content of the Unified model should be chosen in such a way that the anomalies are cancelled. Moreover, some physics should be added in order to provide "unnecessary" fermions with the masses well above 1 TeV scale. Besides, one must construct the unambiguous theory in such a way that at low energies the parallel transporters indeed have the form (58). Let us suppose that this program is fulfilled. Then all parallel transporters

in the theory are invariant under (6) in a natural way. The gauge group $SU(10)$ is simply connected. That's why the Unified theory should contain monopole - like topological objects. As it was already mentioned, their masses and magnetic fluxes are related essentially to the \mathcal{Z} symmetry.

XI. DYNAMICS

Now let us consider the dynamics of Technicolor. It is related in a usual way to the number of fermions N_f . Namely, the beta - function in one loop approximation has the form:

$$\beta_{SU(K)}(\alpha) = -\frac{11K - 2N_f}{6\pi}\alpha^2, \quad (60)$$

where $\alpha = \frac{g_{SU(K)}^2}{4\pi}$.

If $N_f < \frac{11}{2}K$, the one loop calculation indicates asymptotic freedom. The two - loop calculations [10] indicate that the chiral symmetry breaking occurs at

$$N_f < N_c \sim K \frac{100K^2 - 66}{25K^2 - 15} \sim 4K \quad (61)$$

This is required for the appearance of gauge boson masses.

In our model we have three generations of Farhi - Susskind technifermions. Therefore, their number is $24 > 4N_{TC} = 16$. However, it is important that only such technifermions enter (61), the masses of which are of the order of Λ_{TC} and smaller (Λ_{TC} is the $SU(4)$ analogue of Λ_{QCD}). Therefore, we suppose that the masses of the third generation technifermions and, probably, the masses of some of the second generation technifermions are essentially larger, than the Technicolor scale. We also assume that the masses of the fermions that carry the indices of higher Hypercolor groups are essentially larger than the Technicolor scale. So, they do not affect the Technicolor dynamics. Thus the $SU(4)$ interactions lead to the chiral symmetry breaking and provide W and Z bosons with their masses.

If the number of fermions approach $N_c \sim 4N_{TC}$, then the behavior of the model becomes close to conformal. In this case the effective charge becomes walking instead of running [24]. So, in our case (two generations of fermions for $N_{TC} = 4$) the behavior of the technicolor may be close to conformal. It may not be conformal if the fermions of the second generation are also extra massive.

As for the higher Hypercolor groups, already for $SU(5)$ interactions the number of the first generation hyperfermions (fermions carrying $SU(5)$ index) is $2(1 + 3 + 4 + 12) = 40 > \frac{55}{2} = 27.5$. We suppose their masses are close to each other. That's why the Hypercolor forces at $K > 4$ are not asymptotic free, and do not confine. As a result the Landau pole is present in their effective charges. This means that our model does not have a rigorous continuum limit, and should be considered as a finite cutoff model. At the energies of the order of this cutoff the new theory should appear that incorporates the Hypercolor tower as an effective low energy theory. In principle, this scale may be extremely large, even of the order of Plank mass depending on the value of $g_{SU(K)}^2$ at low energies. Very roughly this scale (as given by the $SU(5)$ effective charge) can be estimated as

$$\Lambda_h = e^{\frac{6\pi}{(2N_f - 55)\alpha_{SU(5)}(1 \text{ TeV})}} \text{ TeV} \quad (62)$$

Say, if three generations are involved, and $\alpha_{SU(5)}(1 \text{ TeV}) = \frac{1}{300}$, then the Landau pole occurs in the $SU(5)$ gauge coupling at $\Lambda_h \sim 10^{13} \text{ TeV}$.

XII. CONCLUSIONS

In this paper we present the construction of the ultraviolet completion of the Standard Model. This completion is organized as a tower of Hypercolor gauge theories with the common gauge group $G = \dots \otimes SU(5) \otimes SU(4) \otimes SU(3) \otimes SU(2) \otimes U(1)/\mathcal{Z}$. The fermions of the model may carry indices from the fundamental representation of any $SU(N)$ subgroup of the gauge group G . (Index of each representation may appear only once.) In addition we require that the $SU(2)$ subgroup acts only on the left - handed spinors. Then the only uncertainty is the hypercharge assignment. In order to fix the hypercharges of the fermions we first suppose that there exists a one - to one correspondence between the left - handed and the right handed fermions. This correspondence is related to parity conjugation. The definition of the parity conjugation uses an auxiliary $SU(2)$ field. The unitary gauge can always be fixed, which gives to the

left - handed doublets their conventional Standard Model form. So, for any set of $SU(N)$ indices there exist two right - handed fermions and one left - handed fermion. Next, in order to fix their hypercharges we require that the Z_6 symmetry of the Standard Model is continued to the Hypercolor tower. We choose this continuation in the form (6). The resulting discrete symmetry \mathcal{Z} fixes hypercharge of each hyperfermion up to an arbitrary integer number. We prove that an additional constraint may be imposed on these integer numbers such that the chiral anomaly is absent even if the sequence (1) is ended at any rang of the Hypercolor $SU(N)$ subgroups.

The main reason why we apply the additional \mathcal{Z} symmetry to the construction of the model is that we guess the Z_6 symmetry of the Standard Model cannot appear accidentally, and it should be continued in a certain way to the more fundamental theory. Of course, our choice (6) is just one of the possible ways of this continuation. Besides, we may suppose that there exists the more fundamental theory that has our Hypercolor tower as a low energy approximation. Then, its monopole content has a deep relation to the discrete \mathcal{Z} symmetry. Namely, all Hypercolor subgroups are involved in the formation of monopole configurations if the additional \mathcal{Z} symmetry is present.

The dynamics of the theory is organized in such a way that the $SU(4)$ interactions are confining, provide chiral symmetry breaking, and give rise to the masses of W and Z bosons. In order to provide necessary properties of the $SU(4)$ interactions we suppose that the third generation technifermions (and the hyperfermions of higher Hypercolor groups) have masses much larger than the Technicolor scale ~ 1 TeV. The higher Hypercolor interactions are not confining. The theory admits Landau poles in their effective charges. The correspondent energy scale may, however, be extremely large, in principle, it may be even of the order of Plank mass.

The essential feature of our model is that the fermion mass formation is not related to the transformation of technifermions into the other physical states. We suppose the fermion masses are generated at the energies much higher than the Technicolor scale. In order to incorporate fermion masses to the theory we use the auxiliary scalar $SU(2)$ field. The action does not contain dynamical term with the derivatives of this field. The only place in the action, where this field appears is the fermion mass term. That's why this auxiliary field does not cause the well known problems of the usual Standard Model Higgs sector with the dynamical scalar field.

The Hypercolor model described in this paper may be related to the following picture of fundamental forces at the energies above 1 TeV. We can consider an analogy to the Condensed Matter systems, where no detailed description of microscopic physics is known. Nevertheless, in such systems the simple excitations and their interactions may be described in an elegant and simple way. Symmetry properties play an important role in such a description. Our tower of Hypercolor gauge groups may play the role of such effective description of an unknown microscopic physics, that is to appear above 1 TeV. From this point of view the appearance of all gauge groups $SU(N)$ with any N and all possible fermions from their fundamental representations is quite natural.

Acknowledgments

This work was partly supported by RFBR grants 09-02-00338, 08-02-00661, and 07-02-00237, by Grant for leading scientific schools 679.2008.2, by Federal Program of the Russian Ministry of Industry, Science and Technology No 40.052.1.1.1112.

-
- [1] Christopher T. Hill, Elizabeth H. Simmons, Phys.Rept. 381 (2003) 235-402; Erratum-ibid. 390 (2004) 553-554
Kenneth Lane, hep-ph/0202255
R. Sekhar Chivukula, hep-ph/0011264
 - [2] Thomas Appelquist, Neil Christensen, Maurizio Piai, Robert Shrock, Phys.Rev. D70 (2004) 093010
Adam Martin, Kenneth Lane, Phys.Rev. D71 (2005) 015011
Thomas Appelquist, Maurizio Piai, Robert Shrock, Phys.Rev. D69 (2004) 015002
Robert Shrock, hep-ph/0703050
Adam Martin, Kenneth Lane, Phys.Rev. D71 (2005) 015011
 - [3] Maxim Perelstein, Prog.Part.Nucl.Phys.58:247-291,2007
 - [4] Jan Kalinowski, Int.J.Mod.Phys.A22:5920-5934,2007
 - [5] Thomas Appelquist, Ho-Ung Yee, Phys.Rev. D67 (2003) 055002
 - [6] A. Nicolaidis, N.G. Sanchez, Mod.Phys.Lett. A20 (2005) 1203-1208
 - [7] J.A. Casas, J.R. Espinosa, I. Hidalgo, hep-ph/0607279
F. del Aguila, R. Pittau, Acta Phys.Polon. B35 (2004) 2767-2780
 - [8] A.I.Veselov, M.A.Zubkov, JHEP 0812:109,2008
 - [9] B.L.G. Bakker, A.I. Veselov, and M.A. Zubkov, J. Phys. G: Nucl. Part. Phys. 36 (2009) 075008.
 - [10] Thomas Appelquist, John Terning, L.C.R. Wijewardhana, Phys.Rev.Lett. 79 (1997) 2767-2770
Neil D. Christensen, Robert Shrock, Phys.Rev. D72 (2005) 035013

- Masafumi Kurachi, Robert Shrock, Koichi Yamawaki, hep-ph/0704.3481
T. Appelquist, F. Sannino, Phys.Rev. D59 (1999) 067702
T. Appelquist, P.S. Rodrigues da Silva, F. Sannino, Phys.Rev. D60 (1999) 116007
- [11] R. Foadi, M.T. Frandsen, T. A. Rytto, F. Sannino, arXiv:0706.1696
Sven Bjarke Gudnason, Chris Kouvaris, Francesco Sannino, Phys.Rev. D73 (2006) 115003
D.D. Dietrich (NBI), F. Sannino (NBI), K. Tuominen, Phys.Rev. D72 (2005) 055001
- [12] F. Sannino, arXiv:0804.0182
N. Evans and F. Sannino, arXiv:hep-ph/0512080.
- [13] E.Farhi, L.Susskind, Phys.Rev.D 20, 3404, 1979
- [14] C.Gardner, J.Harvey, Phys. Rev. Lett. **52** (1984) 879
Tanmay Vachaspati, Phys.Rev.Lett. 76 (1996) 188-191
Hong Liu, Tanmay Vachaspati, Phys.Rev. D56 (1997) 1300-1312
- [15] K.S. Babu, I. Gogoladze, and K. Wang, Phys. Lett. B **570**, 32 (2003);
K.S. Babu, I. Gogoladze, and K. Wang, Nucl. Phys. B **660**, 322 (2003);
- [16] B.L.G. Bakker, A.I. Veselov, and M.A. Zubkov, Phys. Lett. B **583**, 379 (2004);
- [17] M.A. Zubkov, Phys. Lett. B **649**, 91 (2007), Erratum-ibid.B655:91,2007.
- [18] M.A.Zubkov, arXiv:0707.0731, ITEP-LAT/2007-11, Physics Letters B674 (2009), 325-329
- [19] J.C.Pati, S.Radjpoot, A.Salam, Phys. Rev. **D 17**, 131 (1978)
- [20] S.Weinberg, "The Quantum Theory of Fields", Cambridge, University press, 2001
- [21] J. Preskill, Nucl. Phys. B177, 21 (1981); M. E. Peskin, Nucl. Phys. B175, 197 (1980).
- [22] Gerard 't Hooft, Nucl.Phys.B79:276-284,1974
Alexander M. Polyakov, JETP Lett.20:194-195,1974, Pisma Zh.Eksp.Teor.Fiz.20:430-433,1974
- [23] Andrzej J. Buras, P.Q. Hung, Phys. Rev. D **68**, 035015 (2003).
Andrzej J. Buras, P.Q. Hung, Ngoc-Khanh Tran, Anton Poschenrieder, and Elmar Wyzomirski, Nucl.Phys. B **699**, 253 (2004);
Mehrddad Adibzadeh and P.Q. Hung, hep-ph/0705.1154.
- [24] Thomas Appelquist, Anuradha Ratnaweera, John Terning, L. C. R. Wijewardhana, Phys.Rev. D58 (1998) 105017
- [25] Nevertheless, see [12], where the way to solve the problem with the t - quark is suggested.
- [26] The configurations of this kind were considered, for example, in [14], where they represent fundamental monopoles of the $SU(5)$ unified model.